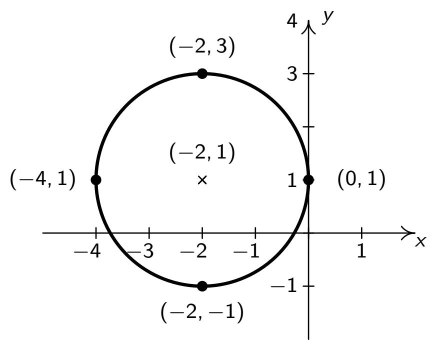
CIRCLES

1. (a) Rewriting $(x+2)^2+(y-1)^2=4$ as $(x-(-2))^2+(y-1)^2=(2)^2$, we identify $h=-2$, $k=1$ and $r=2$. Thus we have a circle centered at $(-2,1)$ with a radius of 2.

   To help us create the graph, we start from the center $(-2,1)$ and move two units to the left and two units up and down to the right to identify four points on the graph. (Note the center of the circle is *not* on the graph of the circle - we mark it for an '$\times$' for reference.) We get $(-2-2,1)=(-4,1)$, $(-2+2,1)=(0,1)$, $(-2,1+2)=(-2,3)$ and $(-2,1-2)=(-2,-1)$.



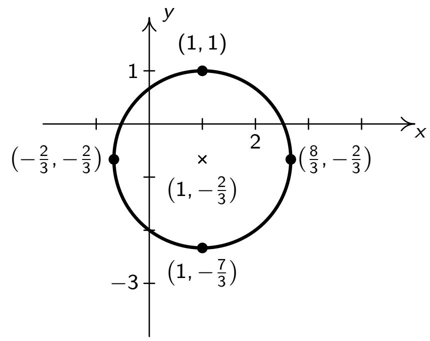
The graph of $(x+2)^2+(y-1)^2=4$

   (b) To graph $3x^2-6x+3y^2+4y-4=0$, we first need to put the equation into standard form. To that end, we complete the square on both the x and y terms and collect the constants to the other side of the equation as demonstrated below.

$$\begin{array}{rcll}
3x^2-6x+3y^2+4y-4 & = & 0 & \\
3x^2-6x+3y^2+4y & = & 4 & \text{Add 4 to both sides.} \\
3\left(x^2-2x\right)+3\left(y^2+\frac{4}{3}y\right) & = & 4 & \text{Factor out leading coefficients.} \\
3\left(x^2-2x+\underline{1}\right)+3\left(y^2+\frac{4}{3}y+\underline{\underline{\frac{4}{9}}}\right) & = & 4+3\underline{(1)}+3\underline{\underline{\left(\frac{4}{9}\right)}} & \text{Complete the Square in } x,\ y. \\
3(x-1)^2+3\left(y+\frac{2}{3}\right)^2 & = & \frac{25}{3} & \text{Factor.} \\
(x-1)^2+\left(y+\frac{2}{3}\right)^2 & = & \frac{25}{9} & \text{Divide both sides by 3.} \\
(x-1)^2+\left(y-\left(-\frac{2}{3}\right)\right)^2 & = & \left(\frac{5}{3}\right)^2 & \text{Rewrite.}
\end{array}$$

   We identify $h=1$, $k=-\frac{2}{3}$, and $r=\frac{5}{3}$. Hence, we have a circle with center $\left(1,-\frac{2}{3}\right)$ and radius $\frac{5}{3}$.

We find four points on the circle by starting at the center $\left(1, -\frac{2}{3}\right)$ and moving up, down, to the left, and to the right $\frac{5}{3}$ units. Doing so gives the points: $\left(-\frac{2}{3}, -\frac{2}{3}\right)$, $\left(\frac{8}{3}, -\frac{2}{3}\right)$, $(1, 1)$, and $\left(1, -\frac{7}{3}\right)$.



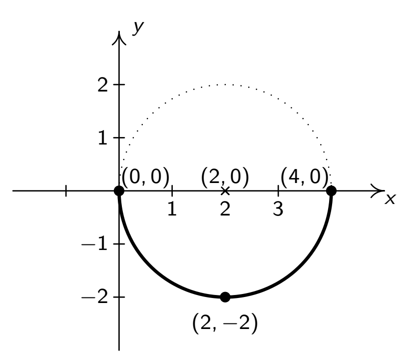
The graph of $3x^2 - 6x + 3y^2 + 4y - 4 = 0$

2. To graph $f(x) = -\sqrt{4x - x^2}$ means we graph the equation $y = f(x)$ which is $y = -\sqrt{4x - x^2}$.

   We can square both sides and put the result into the standard form of a circle:

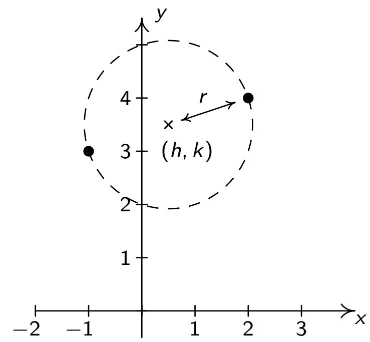
$$
\begin{array}{rcll}
y & = & -\sqrt{4x - x^2} & \\
y^2 & = & \left(-\sqrt{4x - x^2}\right)^2 & \text{square both sides} \\
y^2 & = & 4x - x^2 & \\
x^2 - 4x + y^2 & = & 0 & \text{rewrite} \\
(x - 4x + \underline{\underline{4}}) + y^2 & = & 0 + \underline{\underline{4}} & \text{Complete the Square.} \\
(x-2)^2 + y^2 & = & 4 & \text{Factor.} \\
(x-2)^2 + (y-0)^2 & = & (2)^2 & \text{Rewrite.}
\end{array}
$$

   With $h = 2$, $k = 0$ and $r = 2$, we know the graph of $(x - 2)^2 + y^2 = 4$ is a circle of radius 2 centered at $(2, 0)$. However, the graph we want isn't the *entire* circle.(For one thing, the graph of a circle fails the Vertical Line Test so it does not represent y as a function of x in this case.) Indeed, we want the graph of $y = -\sqrt{4x - x^2}$. Because of the '$-$ ', we want the *lower* semicircle, as seen below.



The graph of $f(x) = -\sqrt{4x - x^2}$.

3. (a) We recall that a diameter of a circle is a line segment containing the center and two points on the circle. We plot the data given to us below to help us visualize the situation.



Since the given points are endpoints of a diameter, we know their midpoint (h,k) is the center of the circle. Likewise, the diameter of the circle is the distance between the given points, so we can find the radius of the circle by taking half of this distance.

$$
\begin{aligned}
(h,k) &= \left(\frac{x_0+x_1}{2}, \frac{y_0+y_1}{2}\right) \\
&= \left(\frac{-1+2}{2}, \frac{3+4}{2}\right) \\
&= \left(\frac{1}{2}, \frac{7}{2}\right)
\end{aligned}
\qquad
\begin{aligned}
r &= \frac{1}{2}\sqrt{(x_1-x_0)^2+(y_1-y_0)^2} \\
&= \frac{1}{2}\sqrt{(2-(-1))^2+(4-3)^2} \\
&= \frac{1}{2}\sqrt{3^2+1^2} \\
&= \frac{\sqrt{10}}{2}
\end{aligned}
$$

Finally, since $\left(\frac{\sqrt{10}}{2}\right)^2 = \frac{10}{4} = \frac{5}{2}$, our answer becomes $\left(x-\frac{1}{2}\right)^2 + \left(y-\frac{7}{2}\right)^2 = \frac{5}{2}$

(b) From the graph given to us, we are safe to assume the center of the circle is $(-2,2)$ since the circle appears to be *tangent* to the coordinate axes at $(-2,0)$ and $(0,2)$. Moreover, since the distance from $(-2,2)$ to either of $(-2,0)$ or $(0,2)$ is 2, the radius of the circle is 2. Hence, our answer is $(x-(-2))^2+(y-2)^2=(2)^2$ or $(x+2)^2+(y-2)^2=4$.